Kennedy Baptist College

Semester One Examination, 2023

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 3

If required by your examination administrator, please place your student identification label in this box

Section One: Calculator-free

WA student number: In figures | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	48	35
Section Two: Calculator-assumed	12	12	100	90	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
   Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
1	7	
2	6	
3	7	
4	7	
5	7	
6	7	
7	7	
S1 Total	48	
S1 Wt (×0.7292)	35%	
S2 Wt	65%	
Total	100%	

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page

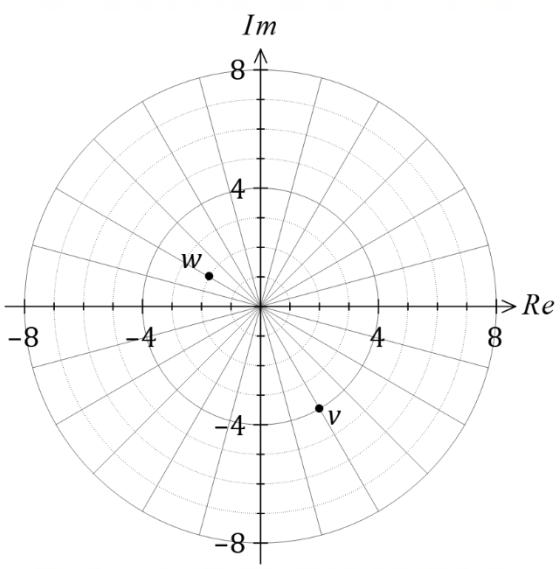
Section One: Calculator-free **35% (48 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(7 marks)**

The diagram shows the complex numbers v and w in the Argand plane.



(a) Express v in

(i) polar form. (1 mark)

(ii) Cartesian form. (1 mark)

(b) Plot and label the following complex numbers on the diagram above:

(i) $z_1 = iv$. (1 mark)

(ii) $z_2 = vw$. (2 marks)

(iii) $z_3 = v - 2w$. (2 marks)

Question 2 **(6 marks)**

The Cartesian equations for three planes are $x - y - z = 2,\ 2x - y + z = 7$ and $3x + y + z = 2$.

(a) Show that none of these planes is parallel to another. (2 marks)

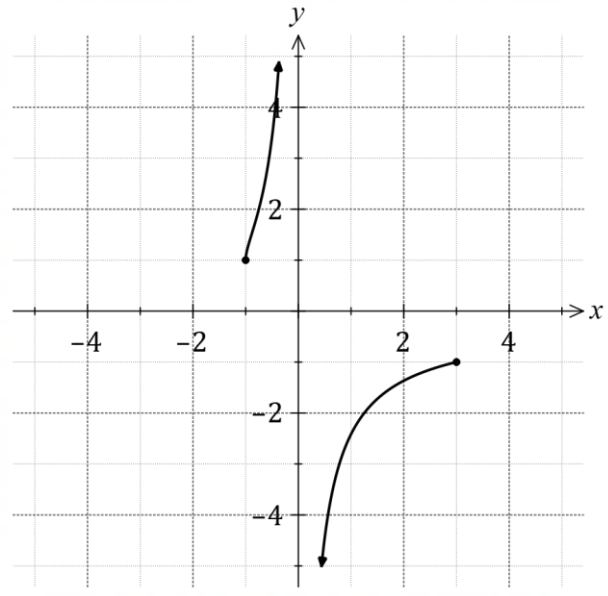
(b) Solve the three equations simultaneously. (3 marks)

(c) State the geometric interpretation of the solution obtained in part (b). (1 mark)

See next page

Question 3 **(7 marks)**

The diagram shows the graph of $y = f(x)$, where $f(x) = \dfrac{1}{1-\sqrt{x+1}}$ and the domain of f is restricted to $\{x \in \mathbb{R} \mid -1 \leq x \leq 3,\ x \neq 0\}$.



(a) Explain how to use the graph to estimate a solution to the equation $f^{-1}(x) = 2$. (1 mark)

(b) On the same axes, sketch the graph of $y = f^{-1}(x)$. (2 marks)

(c) Determine a simplified rule for $y = f^{-1}(x)$, stating any domain restriction(s). (4 marks)

See next page

Question 4 **(7 marks)**

The coordinates of three points in space are $L(0, 3, 3), M(-2, 1, -1)$ and $N(-1, 1, 2)$.

(a) Determine the vector equation of the sphere with diameter LM. (3 marks)

(b) Determine the Cartesian equation of the plane that contains all three points. (4 marks)

Question 5 **(7 marks)**

Consider the function $f(z) = z^4 + 4z^3 + 10z^2 + 20z + 25$.

(a) Determine the remainder when $f(z)$ is divided by $z - i$. (1 mark)

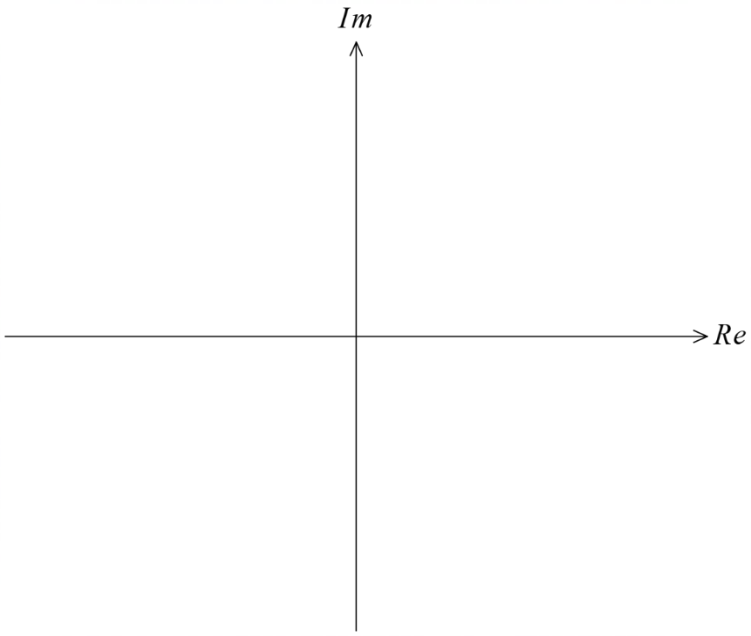
(b) Show that $z - \sqrt{5}i$ is a factor of f. (2 marks)

(c) Solve $f(z) = 0$. (4 marks)

Question 6 **(7 marks)**

(a) Given that $w = \dfrac{\sqrt{3} - i}{1 + i}$, determine the modulus and argument of w. (3 marks)

(b) Sketch the subset of the complex plane determined by $-2|z| = z + \overline{z} - 4$. (4 marks)



Question 7 **(7 marks)**

Consider functions $f(x) = \frac{x^2 + 7}{2}$ and $g(x) = \sqrt{25 - x^2}$.

(a) Explain why f is not a one-to-one function. (1 mark)

(b) State the domain and range of $g(x)$. (2 marks)

(c) Determine the domain and range of $g\big(f(x)\big)$. (4 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____